Real-Time Systems

Part 7: Scheduling
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2. Scheduling Algorithms
   a. Overview
   b. Offline Schedulers
   c. Online Schedulers

3. Schedulability Testing

4. Resources and Resource Access Control
Literature

• Jane W. S. Liu, Real-Time Systems, 2000
• Fridolin Hofmann: Betriebssysteme - Grundkonzepte und Modellvorstellungen, 1991
• Klaus Gresser, Echtzeitnachweis ereignisgesteueter Realzeitsysteme, Dissertation, TUM, 1993

Journals:

• Giorgio C. Buttazzo: Rate Monotonic vs. EDF: Judgement Day (http://www.cas.mcmaster.ca/~downd/rtsj05-rmedf.pdf)
Introduction
Scheduler and Dispatcher

• **Scheduler:**

If a resource is to be used by many consumers, access to the resource has to be coordinated. This resource *allocation* is performed by a *scheduler*.

In computer systems, the term scheduler often refers to the CPU scheduler which controls the allocation of the CPU to *tasks*.

• **Dispatcher:**

While the scheduler plans the CPU allocation, the dispatcher executes the scheduler plan by:

– Switching the context
– Switching to user mode
– Jumping to the proper location in the user program to restart it
We introduce the following model for a task:

- **Release Time (or arrival time) \( r_i \)**
  Earliest time at which task \( i \) is enabled.

- **Start Time \( s_i \)**
  Time at which execution of task starts.

- **Finish Time \( f_i \)**
  Time at which task completes execution.

- **Response Time \( O_i \)**
  Interval between release and finish time.
We introduce the following model for a task:

- **Execution Time** $e_i$
  
  *(remaining execution time $\hat{e}_i$ – see next slide)*
  
  Total time of task execution (does not include durations where the task was blocked).

- **Relative Deadline** $D_i$
  
  *(absolute deadline $d_i$)*
  
  The relative deadline is the maximum tolerated response time.

- **Tardiness**
  
  Measures the deadline violation. $0$ if $f_i \leq d_i$, otherwise $f_i - d_i$
Introduction
Task Model (continued)

- Slack time $t_s$
Introduction
Task Model (continued)

• Preemptable Task
  A task is called **preemptable** if its execution can be suspended.
  – **Fully preemptable**: preemption can occur at any time
  – **Preemption Points**: preemption can only occur at predefined times

• Periodic Task
  A task is called **periodic**, if it is released with a fixed frequency (or period $p$).

• Aperiodic Task
  A task is called **aperiodic**, if it either has a soft deadline or no deadline at all.

• Sporadic Task
  A task is called **sporadic**, if it has a hard deadline but is released at random times.
Introduction
Feasible, Optimal Schedule & Schedulability Test

• Feasible Schedule

A schedule is called **feasible**, if all tasks of the task set \( T_i, i \in \{1,2,\ldots,k\} \) that share the CPU meet their deadlines:

\[
O_i \leq D_i, \forall i \in \{1,2,\ldots,k\}
\]

• Optimal Scheduler

We call a scheduler **optimal** if the algorithm always produces a feasible schedule given that a feasible schedule exists for the task set.

• Schedulability Test

A schedulability test varifies whether a feasible schedule exists for a particular task set.
1. Introduction

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   c. Dynamic Scheduling (Online)

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4. Resources and Resource Access Control
Scheduling Algorithms

Overview

• Static Scheduling (Offline)
  A static scheduling is defined at compile time (offline). All tasks as well as important parameters (e.g. execution times) need to be known a priori.

• Dynamic Scheduling (Online)
  A dynamic scheduling is performed at runtime, based on the current set of active tasks and their resource dependencies.
Scheduling Algorithms
Overview

- **Static Priorities**
  Priority of task depends on task parameters that are known a priori (e.g. deadline or period) and does not change over runtime.

- **Dynamic Priorities**
  Priority of task changes at runtime depending on dynamic parameters (e.g. currently allocated resources).
Scheduling Algorithms
Overview

• **Preemptive**
  A scheduler is called preemptive, if it is able to interrupt the execution of a task and to re-assign the CPU.

• **Non-Preemptive**
  A scheduler is called non-preemptive if it executes a once started task until it finishes or blocks.
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Clock-Driven Scheduling

Notations and Assumptions

• The clock-driven scheduling approach is only applicable if the system is deterministic.

• Assumptions:
  – There are \( n \) periodic tasks in the system.
  – The parameters of all tasks are known a priori.

• Periodic task model notation:
  – There are \( n \) periodic tasks \( T_i \), defined by the 4-tuple:
    \[
    T_i: (\phi_i; p_i; e_i; D_i)
    \]
    where \( \phi_i \) is the phase and \( p_i \) is the period of the periodic task.
  – If the phase is 0, we will omit it.
  – If the period is equal to the relative deadline, we will omit \( D_i \).
Clock-Driven Scheduling
Variable Frame Length Schedule

• A **frame** is the time interval after which the scheduler will be triggered.

• The length of a frame is called the **frame size** $f$.

• **Example of a static scheduler with a variable frame size $f$:**
  
  – Given are four independent periodic tasks that are executed on a single-processor system: $T_i = (p_i, e_i)$

    • $T1 = (4, 1)$
    • $T2 = (5, 1.8)$
    • $T3 = (20, 1)$
    • $T4 = (20, 2)$
Clock-Driven Scheduling
Variable Frame Length Schedule

• **Example (continued):**
  
  – *The hyperperiod* $H$ (*the least common multiple of all* $p_i$*) is 20*
  
  – *A possible static schedule is shown in the following figure (if no task is running the Idle-Task is executed):*

    – *The scheduler is called at times: 0, 1, 2, 3.8, 4, 6, etc.  \( \rightarrow \text{no fixed frame size} \)
**Clock-Driven Scheduling**

**Fixed Frame Length Schedule**

- Ideally, we want to ensure that the cyclic schedule has some desired characteristics, e.g. a constant frame size.

- An optimal, constant frame size can be computed from a task set $T_i$ by taking the following constraints into account (Baker and Shaw, 1988):
  - Constraint 1: The frame size should be smaller than or equal to the relative deadline $D_i$:
    $$ f \leq \min_{1 \leq i \leq k}(D_i) $$
  - Constraint 2: Ideally, the frame size should be large enough to execute the longest task within one single frame:
    $$ f \geq \max_{1 \leq i \leq k}(e_i) $$
Clock-Driven Scheduling
Fixed Frame Length Schedule

- Constraint 3: The hyperperiod $H$ should be an integer multiple of the frame size $f$:

$$ F = \frac{H}{f} \text{ with } F \in N $$

(The relevant frame sizes $f$ can easily be determined by computing all integer factors of the periods of the tasks)

- Constraint 4: The frame size $f$ has to be small enough to ensure that no task misses its deadline (between the release time and the deadline has to fit at least one frame):

$$ 2f - \text{GCD}(p_i, f) \leq D_i $$

(GCD = Greatest Common Divisor)
Clock-Driven Scheduling
Fixed Frame Length Schedule

Constraint 4 – Explanation

\[ t + 2f \leq t'_i + D_i \]
\[ 2f - (t'_i - t) \leq D_i \]

As we are interested in the upper limit of \( f \), we have to compute the smallest possible value of \( (t'_i - t) \) larger than 0: This is the greatest common divisor of \( p_i \) and \( f \):

\[ 2f - \text{GCD}(p_i, f) \leq D_i \]

**Example:**

\( T \) with period 5
Frame size \( f = 3 \)
**Example:**

- **Tasks** \((T_i = (p_i, e_i))\): \(T_1 = (4, 1), T_2 = (5, 1.8), T_3 = (20, 1), T_4 = (20, 2)\)
  
  - **Constraint 1:** \(f \leq 4\)
  - **Constraint 2:** \(f \geq 2\)
  - **Constraint 3:** \(f = \{2, 4, 5, 10, 20\} \Rightarrow \{5, 10, 20\}\) can be ignored due to constraint 1
  - **Constraint 4:**
    - \(f = 2\):
      - \(T_1: 4 - \gcd(4, 2) = 2 \leq 4\) (ok)
      - \(T_2: 4 - \gcd(5, 2) = 3 \leq 5\) (ok)
      - \(T_3: 4 - \gcd(20, 2) = 2 \leq 20\) (ok)
      - \(T_4: 4 - \gcd(20, 2) = 2 \leq 20\) (ok)
    - \(f = 4\):
      - \(T_1: 8 - \gcd(4, 4) = 4 \leq 4\) (ok)
      - \(T_2: 8 - \gcd(5, 4) = 7 \leq 5\) (not ok)

→ Only feasible frame size: \(f = 2\)
Clock-Driven Scheduling
Fixed Frame Length Schedule

- Example (continued):
  - Tasks \((T_i=(p_i, e_i))\): \(T_1=(4,1)\), \(T_2=(5, 1.8)\), \(T_3=(20,1)\), \(T_4=(20,2)\)
Clock-Driven Scheduling
Fixed Frame Length Schedule

• Sometimes the given task set cannot meet the four frame size constraints simultaneously.

• Example:
Consider the task set: $T_i=(p_i, e_i, D_i)$
$T_1=(4,1)$, $T_2=(5,2,7)$, $T_3=(20,5)$
  – To satisfy constraint 1: $f \leq 4$
  – To satisfy constraint 2: $f \geq 5$
  → This is not possible!!!

• Solution: Partition a task into subtasks.
Clock-Driven Scheduling  
Fixed Frame Length Schedule

- E.g. partitioning $T_3 = (20, 5)$ in:
  - $T_{3,1} = (20, 1)$,
  - $T_{3,2} = (20, 3)$ and
  - $T_{3,3} = (20, 1)$

yields a frame size of 4.
Clock-Driven Scheduling
Fixed Frame Length Schedule, Aperiodic Tasks

• Aperiodic tasks are scheduled after all tasks with hard deadline requirements are scheduled.

• To improve the response time of aperiodic tasks, they should be executed before the periodic tasks.

→ This is called slack-stealing
Clock-Driven Scheduling
Fixed Frame Length Schedule, Aperiodic Tasks

- Slack-Stealing Example

\[ A_1 \quad (e_1 = 1.5) \quad A_2 \quad (e_2 = 0.5) \quad A_3 \quad (e_3 = 2) \]

Without aperiodic jobs

Aperiodic Jobs no slack-stealing

Aperiodic Jobs Slack-stealing

Average Response Time of A1, A2 and A3: 4.5

Average Response Time of A1, A2 and A3: 2.5
Clock-Driven Scheduling

**Fixed Frame Length Schedule, Sporadic Tasks**

- Sporadic tasks have, similar to periodic tasks, hard deadlines.
- If more than one sporadic task is waiting, they should be ordered on the Earliest-Deadline-First (EDF) basis.
- Whether a sporadic task $S(d, e)$ is accepted or rejected by the scheduler is determined by an **acceptance** test.

  **Acceptance Test:**
  The sporadic task $S$ is accepted if the accumulated slack times from frame $t$ to $l$
  
  $\sigma_c(t, l)$ is greater than or equal to the execution time of the sporadic task $S(d, e)$.

  $e \leq \sigma_c(t, l)$

  
  $\sigma_c(t, l) = \sigma_t + \sigma_{t+1} + \ldots + \sigma_{l-1} + \sigma_l$
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Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate Monotonic Algorithm

• In the *rate monotonic* (RM) algorithm, task priorities depend on the task rate \(1/p_i\)
  \(\Rightarrow\) the higher the rate, the higher the priority.

• *Example:*

  – Task-Set: \(T_i = (p_i, e_i)\)
    • \(T_1=(4,1) \Rightarrow \text{Priority high}\)
    • \(T_2=(5,2) \Rightarrow \text{Priority medium}\)
    • \(T_3=(20,5) \Rightarrow \text{Priority low}\)
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate Monotonic Algorithm

• **Example:** $T_1=(4,1), T_2=(5,2), T_3=(20,5)$
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Deadline Monotonic Algorithm

• In the **deadline monotonic** (DM) algorithm, task priorities depend on the *relative* task deadline $D_i$

→ the shorter the relative deadline, the higher the priority.

• **Example:**

  – Task-Set: $T_i = (\phi_i, p_i, e_i, D_i)$

    • $T_1 = (50, 50, 25, 100) \rightarrow \text{Priority low}$
    • $T_2 = (0, 62.5, 10, 20) \rightarrow \text{Priority high}$
    • $T_3 = (0, 125, 25, 50) \rightarrow \text{Priority medium}$
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Deadline Monotonic Algorithm

- **Example (continued):** $T_i = (\phi_i, p_i, e_i, D_i)$
  
  $T_1 = (50, 50, 25, 100)$, $T_2 = (0, 62.5, 10, 20)$, $T_3 = (0, 125, 25, 50)$
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate vs. Deadline Monotonic

• Important notes:
  – If the relative deadlines and the periods of all tasks are proportional, the rate and deadline monotonic algorithms are identical.
  – When the relative deadlines are arbitrary, the DM algorithm can sometimes produce a feasible schedule when the RM algorithm fails.
  – The RM algorithm always fails when the DM algorithm fails.
Priority-Driven Scheduling
Periodic Tasks, Static Priorities, Rate vs. Deadline Monotonic

- Previous DM example, scheduled by a RM scheduler:
  - DM resulted in feasible schedule, RM fails.

![Diagram showing scheduling examples](image-url)
Priority-Driven Scheduling

Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

• The Earliest-Deadline-First (EDF) algorithm assigns priorities to tasks according to their absolute deadlines $d_i$.

→ The earlier the deadline, the higher the priority.

• Example:

  – Given task set: $T_i=(p_i, e_i)$

    • $T_1 = (2, 0.9)$
    • $T_2 = (5, 2.3)$
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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![Diagram showing priority-driven scheduling with tasks $T_1$ and $T_2$.]
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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![Chart showing the scheduling process with tasks $T_1$ and $T_2$.]
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

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Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Earliest-Deadline-First (EDF) Algorithm

- Example (continued): $T_1 = (2, 0.9), T_2 = (5, 2.3)$
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

• The Least-Slack-Time-First algorithm assigns priorities to tasks according to their slack time.
  → the smaller the slack time, the higher the priority

• Definition of slack time (recapitulation):

Note:
  – Slack time of currently running processes is constant.
  – Slack time of waiting processes shortens.
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- **Example** \((T_i=(p_i, e_i))\): \(T_1 = (2, 0.8), T_2 = (5, 1.5), T_3 = (5.1, 1.5)\)
- **Slack-Time**: \(t_s = d - t - \hat{e}\)

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Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

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![Diagram showing priority-driven scheduling with slack-time calculations]
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

- **Example** ($T_i=(p_i, e_i)$): $T_1 = (2, 0.8)$, $T_2 = (5, 1.5)$, $T_3 = (5.1, 1.5)$

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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

• Example \( (T_i=(p_i, e_i)) \): \( T_1 = (2, 0.8) \), \( T_2 = (5, 1.5) \), \( T_3 = (5.1, 1.5) \)

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\[
\begin{array}{|c|c|c|c|}
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\text{t} & d / \hat{e} / t_s \\
\hline
    & T_1 & T_2 & T_3 \\
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0 & 2 / 0.8 / 1.2 & 5 / 1.5 / 3.5 & 5.1 / 1.5 / 3.6 \\
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2 & 4 / 0.8 / 1.2 & 5 / 0.3 / 2.7 & 5.1 / 1.5 / 1.6 \\
2.8 & - & 5 / 0.3 / 1.9 & 5.1 / 1.5 / 0.8 \\
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Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Least-Slack-Time-First (LST) Algorithm

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<tr>
<th>(t)</th>
<th>(d / \hat{e} / t_s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(T_1)</td>
</tr>
<tr>
<td>0</td>
<td>2 / 0.8 / 1.2</td>
</tr>
<tr>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>4 / 0.8 / 1.2</td>
</tr>
<tr>
<td>2.8</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>6 / 0.8 / 1.2</td>
</tr>
</tbody>
</table>
Priority-Driven Scheduling
Periodic Tasks, Dynamic Priorities, Summary EDF and LST

• Both, EDF and LST are optimal if:
  – Preemption of tasks is allowed
  – Tasks do not contend for resources
  – A single processor system is used

• EDF does not require knowledge of execution times, LST does
  → huge drawback
1. Introduction

2. Scheduling Algorithms
   a. Overview
   b. Offline Schedulers
   c. Online Schedulers

3. Schedulability Testing

4. Resources and Resource Access Control
Schedulability Testing
Introduction

• A test to validate that a given set of tasks can meet its hard deadlines when scheduled according to a specific scheduling algorithm is called *schedulability* test.
Schedulability Testing
DM and RM Algorithms

- A task set of $n$ tasks can be feasibly scheduled on one processor by the RM algorithm if the following utilization condition holds (Liu und Layland 1973):
  \[ U = \sum_{i=1}^{n} \frac{e_i}{p_i} \leq n(2^{1/n} - 1) \]

- Note: The tasks have to be:
  - independent,
  - preemptable, and
  - periodic.

*Recapitulation:* If the relative deadlines of all task in a given task set are proportional to the periods, the DM algorithm is identical to the RM algorithm and the above condition can also be used to perform a schedulability test for the DM algorithm.
Schedulability Testing
DM and RM Algorithms

• **Example:**

<table>
<thead>
<tr>
<th>Task</th>
<th>$p_i$</th>
<th>$e_i$</th>
<th>$u_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>1.25</td>
<td>0.1</td>
<td>0.08</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>1.75</td>
<td>0.07</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>2.0</td>
<td>0.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Sum: 0.62**

*Total utilization $U=0.62 \leq 0.743 \rightarrow$ task set can be feasibly scheduled by the RM algorithm.*
Schedulability Testing
DM and RM Algorithms

• **Important:**
  *The presented condition is not a necessary condition!!!*

  → *Even if the utilization of a task set exceeds the condition, a feasible RM schedule might exist.*

• A schedulability test of such a task set, scheduled by a fixed-priority algorithm, can be performed by the **time-demand analysis**.
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

• For a sorted task set $T_i$ (i.e. $T_0 =$ task with highest priority, $T_i =$ task with lowest priority), we can perform a time-demand analysis, by (Lehoczky et al., 1989)

1. computing the time-demand of all tasks $T_i$, according to:

$$w_i(t) = e_i + \sum_{k=1}^{i-1} \left\lceil \frac{t}{p_k} \right\rceil e_k \quad \text{for} \ 0 < t \leq p_i$$

2. checking whether the inequality

$$w_i(t) \leq t$$

is satisfied for values of $t$ that are equal to

$$t = j p_k; k = 1, 2, \ldots, i; j = 1, 2, \ldots, \left\lfloor \min(p_i, D_i) / p_k \right\rfloor$$

If this inequality is satisfied at one of these instants, $T_i$ is schedulable.
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

• Example:
  \( T_1 = (\phi_1, 3, 1); T_2 = (\phi_2, 5, 1.5), T_3 = (\phi_3, 7, 1.25), T_4 = (\phi_4, 9, 0.5) \)
  
  - \( w_1 \):
    - \( w_1(3) = 1 \leq 3 \Rightarrow OK \)
  
  - \( w_2 \):
    - \( w_2(3) = 1.5 + 1 = 2.5 \leq 3 \Rightarrow OK \)
  
  - \( w_3 \):
    - \( w_3(3) = 1.25 + 1 + 1.5 = 3.75 > 3 \Rightarrow Not \, OK \)
    - \( w_3(5) = 1.25 + 2 + 1.5 = 4.75 \leq 5 \Rightarrow OK \)
  
  - \( w_4 \):
    - \( w_4(3) = 0.5 + 1 + 1.5 + 1.25 = 4.25 > 3 \Rightarrow Not \, OK \)
    - \( w_4(5) = 0.5 + 2 + 1.5 + 1.25 = 5.25 \leq 5 \Rightarrow Not \, OK \)
    - \( w_4(6) = 0.5 + 2 + 3 + 1.25 = 6.75 > 6 \Rightarrow Not \, OK \)
    - \( w_4(7) = 0.5 + 3 + 3 + 1.25 = 7.75 > 7 \Rightarrow Not \, OK \)
    - \( w_4(8) = 0.5 + 3 + 3 + 2.5 = 9 \leq 9 \Rightarrow OK \)
Schedulability Testing
Time-Demand Analysis for Fixed-Priority Algorithms

- Example (continued):

Graphical demonstration of time-demand analysis
Schedulability Testing
EDF Algorithm

• Task density:
  \[ \text{density}_k = \frac{e_k}{\min(D_k, p_k)} \]

• A set of
  – independent,
  – periodic, and
  – preemptable

  tasks can be *feasibly* scheduled by the EDF algorithm on one processor if the task set density is less or equal to 1:

  \[ \sum_{k=1}^{n} \frac{e_k}{\min(D_k, p_k)} \leq 1 \]

**Note:** This is only a sufficient condition. Even if inequality is not satisfied, a feasible schedule might exist.
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3. Schedulability Testing

4. Resources and Resource Access Control
Resources and Resource Access Control

Introduction

• If resources can only be used in a mutual exclusive manner, resource contentions occur that can lead to system failures.

• Effects of resource contentions:
  – Priority Inversions
  – Deadlocks
Resources and Resource Access Control
Effects of Resource Contention: Priority Inversion

- The phenomenon that a lower-priority task blocks a higher-priority task is called *priority inversion*.
Resources and Resource Access Control
Effects of Resource Contention: Uncontrolled Priority Inversion

- **Uncontrolled (or Unbounded) Priority Inversion**
  A medium priority task can block a high priority task forever.

Uncontrolled priority inversion can only occur if the task set contains more than 2 tasks.
Resources and Resource Access Control
Effects of Resource Contention: Deadlock

- Consider two tasks $T_1$ and $T_2$ and two resources $R_1$ and $R_2$.
  - $T_1$ holds $R_1$, requests $R_2$
  - $T_2$ holds $R_2$, requests $R_1$
  $\Rightarrow$ Deadlock
Resources and Resource Access Control

Nonpreemptive Critical Section (NPCS) Protocol

- Simple way to control access to a resource is to schedule all critical sections nonpreemptively:

  If a task request a resource, it is always allocated the resource and executes with the highest priority.

  ➔ This protocol is called the Nonpreemptive Critical Section (NPCS) protocol

- As no preemption takes place, no deadlock or priority inversion can occur!!!

- Shortcoming: Every task can be blocked by every lower-priority task, even if there is no resource conflict.
Resources and Resource Access Control
Basic Priority Inheritance Protocol (BPIP)

• The basic priority inheritance protocol (BPIP) prevents uncontrolled priority inversions but not deadlocks.

→ This is achieved by raising the current priority $\pi_l(t)$ of a lower-priority task to a higher (inherited) priority $\pi_h(t)$ of another task.

• BPIP rules:
  – Scheduling Rule: Ready tasks are scheduled preemptively in a priority-driven manner according to their current priorities. At the release time, the current priority $\pi(t)$ is equal to the assigned priority (the priority determined by the scheduling algorithm).
Resources and Resource Access Control
Basic Priority Inheritance Protocol (BPIP)

• BPIP rules (continued):
  
  – Allocation Rule: When a task $T$ requests a resource $R$ at time $t$,
    
    a) if $R$ is free, $R$ is allocated to $T$ until $T$ releases the resource, and
    b) if $R$ is not free, the request is denied and $T$ is blocked.
  
  – Priority-Inheritance Rule: When the requesting task $T$ becomes blocked, the task $T_i$ which blocks $T$ inherits the current priority of $T$ until it releases the resource. At that time, the priority of $T_i$ returns to the value it had at the time when it acquired $R$. 

### Resources and Resource Access Control

#### Basic Priority Inheritance Protocol (BPIP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T5 executes with priority 5</td>
</tr>
<tr>
<td>1</td>
<td>T5 is granted resource “black”</td>
</tr>
<tr>
<td>2</td>
<td>T4 released, preempts T5</td>
</tr>
<tr>
<td>3</td>
<td>T4 is granted resource “dotted”</td>
</tr>
<tr>
<td>4</td>
<td>T3 released, preempts T4</td>
</tr>
<tr>
<td>5</td>
<td>T2 released, preempts T3</td>
</tr>
<tr>
<td>6</td>
<td>T2 requests resource “black”, T5 inherits priority of T2 and executes</td>
</tr>
<tr>
<td>7</td>
<td>T1 released, preempts T5</td>
</tr>
</tbody>
</table>

---

### Diagram

- **T1**: Event at time 0
- **T2**: Event at time 1
- **T3**: Event at time 2
- **T4**: Event at time 3
- **T5**: Event at time 4
- **T2**: Event at time 5
- **T1**: Event at time 6
Resources and Resource Access Control
Basic Priority Inheritance Protocol (BPIP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>T1 requests resource “dotted”, T4 inherits priority of T1</td>
</tr>
<tr>
<td>9</td>
<td>T4 requests resource “black”, T5 inherits priority and executes</td>
</tr>
<tr>
<td>11</td>
<td>T5 releases resource “black”, T4 continues</td>
</tr>
<tr>
<td>13</td>
<td>T4 releases resource “dotted”, T1 acquires resource “dotted” and continues</td>
</tr>
<tr>
<td>15</td>
<td>T1 completes, T2 is granted resource “black” and executes</td>
</tr>
<tr>
<td>17</td>
<td>T2 completes, afterwards T3, T4 and T5 execute and complete</td>
</tr>
</tbody>
</table>
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

• The basic priority ceiling protocol (BPCP) extends the BPIP to prevent deadlocks and to further reduce the blocking time.

• **Priority Ceiling**: The priority ceiling $\Pi(R_i)$ of a resource $R_i$ is the highest priority of all the tasks that require $R_i$.
  
  – *Example (based on previous slide):* $\Pi(B) = 2$, $\Pi(D) = 1$

• **Current Priority Ceiling (or simply ceiling)**: The ceiling $\hat{\Pi}(t)$ is equal to the highest priority ceiling of the resources currently in use. If all resources are free, the ceiling is equal to $\Omega$, a non-existing priority lower than any other priority.
  
  – *Example (based on previous slide):*
    
    • In (1,3], resource „black“ is used; hence the ceiling is 2
    • In (3,13], resource „dotted“ is used; hence the ceiling is 1
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

• BPCP rules:
  – *Scheduling Rule:*
    a) At its release time, the current task priority $\pi(t)$ is equal to its assigned priority.
    b) Every ready task is scheduled preemptively and in a priority-driven manner, depending on its current priority $\pi(t)$.
  – *Allocation rule:*
    Whenever a task $T$ requests a resource $R$ at time $t$, one of the following conditions occurs:
    a) $R$ is held by another task $\Rightarrow T$ blocks
    b) $R$ is free
       a) If the priority $\pi(t)$ of $T$ is higher than the current priority ceiling, $R$ is allocated to $T$.
       b) If the priority of $T$ is not higher than the ceiling, $R$ is allocated to $T$ only if $T$ is holding the resource whose priority ceiling is equal to the ceiling; otherwise $T$ blocks.
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP)

- BPCP rules:
  - *Priority Inheritance Rule*: When $T$ becomes blocked, the task $T_i$ that blocks $T$ inherits the current priority of $T$. $T_i$ executes at its inherited priority until the time when it releases every resource whose priority ceiling is equal to or higher than the priority of $T$; at that time, the priority of $T_i$ returns to the value it had when it was granted the resource.
## Resources and Resource Access Control

### Basic Priority Ceiling Protocol (BPCP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>T5 executes with priority 5</td>
</tr>
<tr>
<td>1</td>
<td>T5 is granted resource “black”</td>
</tr>
<tr>
<td>2</td>
<td>T4 released, preempts T5</td>
</tr>
<tr>
<td>3</td>
<td>T4 requests resource “dotted”, but the request is denied (priority of T4 lower than current ceiling). T5 inherits priority of T4 and executes at priority 4.</td>
</tr>
<tr>
<td>4</td>
<td>T3 released, preempts T5</td>
</tr>
<tr>
<td>5</td>
<td>T2 released, preempts T3</td>
</tr>
<tr>
<td>6</td>
<td>T2 requests resource “black” and becomes blocked by T5; T5 inherits priority 2</td>
</tr>
</tbody>
</table>

---

**Diagram:**

- **T1:** Event with priority 1
- **T2:** Event with priority 2
- **T3:** Event with priority 3
- **T4:** Event with priority 4
- **T5:** Event with priority 5

- **Resource Access Control:**
  - Basic Priority Ceiling Protocol (BPCP)
  - Example scenario
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>T1 becomes ready and preempts T5</td>
</tr>
<tr>
<td>8</td>
<td>T1 requests resource “dotted”; Priority of T1 higher than ceiling, resource request is granted</td>
</tr>
<tr>
<td>10</td>
<td>T3 and T5 are ready, T5 has higher priority (2) and executes</td>
</tr>
<tr>
<td>11</td>
<td>T5 releases “black” and its priority returns to 5; the ceiling drops to ω; T2 unblocks, allocates „black“ and executes</td>
</tr>
<tr>
<td>14</td>
<td>J4 is granted “dotted” as its priority is higher than the ceiling</td>
</tr>
</tbody>
</table>
Resources and Resource Access Control
Basic Priority Ceiling Protocol (BPCP), Example

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>T4 requests “black”, which is free. The priority of T4 is lower than the ceiling, but T4 is holding the resource whose priority ceiling is equal to the current ceiling (“dotted”).</td>
</tr>
</tbody>
</table>